New Horizons in Globular Cluster Astronomy
ASP Conference Series, Vol. 296, 2003
G. Piotto, G. Meylan, S. G. Djorgovski, and M. Riello, eds.

Dynamical Models with Stellar Evolution and Binaries

Jarrod R. Hurley, Michael M. Shara

American Museum of Natural History, Department of Astrophysics, Central Park West at 79th Street, New York, NY 10024, USA

Sverre J. Aarseth, Christopher A. Tout

Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK

Onno R. Pols

Astronomical Institute, Utrecht University, Postbus 80000, 3508 TA Utrecht, the Netherlands

Abstract. The recent availability of the powerful GRAPE-6 hardware, designed specifically to tackle the gravitational N-body problem, has coincided with the production of realistic N-body software that includes a detailed treatment of stellar and binary evolution. In light of GRAPE-6 simulations performed with the `NBODY4` code, the role that binaries and the cluster environment play in modifying the stellar populations of a star cluster is discussed. Particular emphasis is placed on the behaviour and appearance of the white dwarf populations in star clusters.

1. Introduction

The merits and shortcomings of the various methods available for modelling the evolution of star clusters have been discussed in detail elsewhere (Spurzem, these proceedings; see also, Meylan & Heggie 1997) so we choose not to dwell on the details here. Suffice to say that the N-body method has many advantages: it is direct, it lends itself well to the addition of algorithms to treat realistic processes, such as stellar evolution, and it is particularly useful when investigating the stellar populations of clusters because it is possible to follow the individual orbits of the stars. The main disadvantage of the N-body method has been its prohibitively large computational cost but this has now been addressed by the availability of the GRAPE hardware designed specifically for the calculation of Newtonian gravitational forces (Makino & Taiji 1998). The latest incarnation is the GRAPE-6 which operates at a peak speed of 1 Tflop (for a 32-chip production board) and allows simulations of star clusters with $N = 100\,000$ stars to be performed for the first time. Compare this with the situation a decade ago, in the pre-GRAPE era, when the integration of a cluster with $N = 2\,500$ stars, and with < 100 primordial binaries, was a considerable achievement (Heggie & Aarseth 1992).

The inclusion of binaries in star cluster simulations is necessary for a number of reasons, the most obvious being that clusters are observed to have sizeable binary fractions, so neglecting binaries leads to unrealistic models. Importantly, star clusters depend on the presence of binaries in their cores to provide a source of heat against core-collapse and thus ensure their long term survival. Accounting for the finite size of the cluster stars allows for the detection of contact binaries which may deprive the cluster of some of this fuel. Furthermore, modelling of the processes involved in close-binary evolution, and the interaction of these binaries with other cluster stars (or binaries) is also important if we aim to explain the observed excess of exotic objects in star clusters. Blue stragglers and low-mass X-ray binaries are prime examples (Bailyn 1995). An extensive review of the situation regarding binaries in globular clusters from both an observational and theoretical standpoint is provided by Hut et al. (1992).

Implementation of an efficient yet extensive treatment of binary evolution in an N-body code has proven to be a non-trivial exercise – to put it mildly (McMillan, these proceedings; see also Aarseth 1999). Consider that a short-period binary in the core of a cluster that is subject to perturbing forces from nearby stars requires a timestep that is less than its orbital period by at least a factor of 30 for accurate integration. Compare this to the lifetime of a cluster ($\sim$ Gyr) and you see that the existence of even one such binary has the potential to reduce the simulation almost to a standstill. Furthermore, the $1/r^2$ nature of the gravitational force gives rise to large terms when two stars become arbitrarily close – which may be fatal for the simulation. Thankfully a number of algorithmic advances have been made to deal with these problems. The block-timestep scheme allows each particle to be integrated on its own natural dynamical timescale while allowing for a number of particles to be integrated at any one time. Regularization techniques have been developed to remove the singularities involved with compact binaries and few-body configurations, as well as to improve the accuracy of the treatment. However, binaries still remain a major bottleneck in the timely completion of N-body star cluster simulations because most of the operations related to their evolution are performed on the host computer. Generally the GRAPE only sees a binary as the centre-of-mass particle and the tidal corrections to the component stars are applied on the host. Aspects of the evolution such as Roche-lobe overflow, tidal circularization, and magnetic braking are dealt with on the host as necessary. While a simulation of 30 000 single stars takes less than a week to complete with one GRAPE-6 board attached to a fast PC or alpha host, if we include a 50% binary population the CPU time approaches a month.

2. The N-body simulations

We present results from N-body simulations performed with the `NBODY4` code (Aarseth 1999) on the GRAPE-6 special-purpose computers housed at the American Museum of Natural History. The `NBODY4` code accounts for the evolution of single stars and binaries (mass-loss, mass-transfer, mergers, etc.) while modelling all aspects of the dynamical evolution of the cluster (see Hurley et al. 2001, and references therein, for full details). In particular, the single star evolution algorithm adopted by `NBODY4` is that of Hurley, Pols & Tout (2000).

We focus here on the results of an N-body simulation that started with 30 000 stars and a primordial binary frequency, f_{b}, of 50%. For comparison we also performed a simulation with 30 000 stars and no primordial binaries. The initial mass function (IMF) of Kroupa, Tout & Gilmore (1993) was used to assign the masses of single stars and a metallicity of $Z = 0.02$ was assumed. For primordial binaries the total mass of the binary was chosen from the IMF of Kroupa, Tout & Gilmore (1991), since this was not corrected for the effect of binaries, and the component masses were then assigned according to a uniform mass-ratio distribution. Individual stellar masses were restricted to lie within the limits of $0.1 - 50 M_{\odot}$. The orbital separation of each primordial binary was taken from the log-normal distribution within the limits of $6 R_{\odot} - 200\,\mathrm{AU}$, and the orbital eccentricity was taken from a thermal distribution. If we consider the computational effort required for the treatment of perturbed binaries the make-up of the primordial binary population is a prime concern. A binary that is a member of a star cluster is defined as *hard* if the absolute value of its binding energy exceeds the mean kinetic energy of typical cluster stars. As a general rule *soft* binaries will be broken up and hard binaries will become harder as a result of gravitational encounters with other stars and binaries as the cluster evolves. Extremely hard binaries are less likely to experience significant orbital perturbation so it is the primordial binaries near the hard/soft boundary that are most likely to be problematic. The primordial binary population used in these simulations contained 73% hard binaries and 40% of those had binding energies within a factor of 10 of the hard/soft boundary, i.e. these were expensive simulations.

We used a Plummer model in virial equilibrium to set the initial positions and velocities of the stars. Each simulated cluster was assumed to be on a circular orbit within a Keplerian potential with a speed of $220\,\mathrm{km\,s^{-1}}$ at a distance of 8.5 kpc from the Galactic Centre. Stars were removed from the simulation when their distance from the cluster centre exceeded twice the tidal radius defined by this tidal field. All stars were on the zero-age main sequence (MS) when the simulation began and any residual gas from the star formation process was assumed to have already left the cluster. Mass lost from stars during the simulation was simply removed from the cluster, with the cluster potential adjusted accordingly in order to ensure energy conservation.

Each cluster started with a total mass of about $14\,000 M_{\odot}$ and was evolved to an age of 4.5 Gyr. The initial velocity dispersion of the stars was $\sim 3.2\,\mathrm{km\,s^{-1}}$ and the core density was $\sim 500\,\mathrm{stars\,pc^{-3}}$. As a result, our findings are directly applicable to massive open clusters.

3. The Effect of Binaries

Figure 1 compares the simulations with and without primordial binaries by looking at two key cluster properties: the total stellar mass (left panel) and the core density (right panel). The presence of primordial binaries clearly affects the subsequent evolution of a star cluster, as was to be expected.

The primary reason for the higher rate of mass loss from the cluster with primordial binaries is an enhanced rate of escaping stars via velocity kicks obtained in 3-body interactions. After 4 Gyr of evolution the simulation with

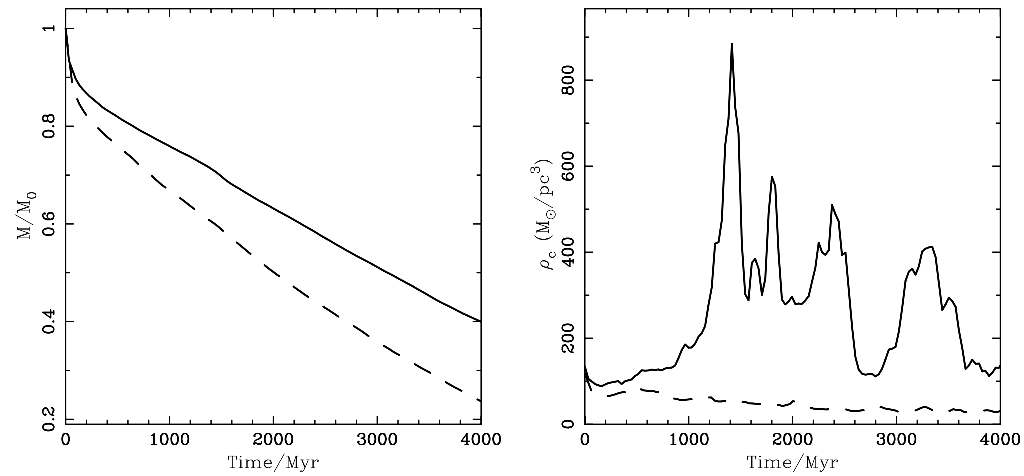

Figure 1. Comparison of two simulations that each started with 30 000 stars: one had no primordial binaries (solid line) and the other had 50% primordial binaries (dashed line). The left panel shows the evolution of the total cluster mass (as a fraction of the initial mass) and the right panel shows the evolution of the stellar mass density in the core of the cluster.

primordial binaries had lost 54% of its initial mass in escaping stars compared with only 40% for the single star simulation. The average velocity of escaping single MS stars was 20% higher for the former simulation. Including binaries in the simulation also creates the possibility of additional mass-loss resulting from non-conservative mass transfer and stellar mergers. This turns out to have negligible effect on the total cluster mass: the inclusion of primordial binaries increased the mass lost owing to stellar and binary evolution processes from 26% to 28% (after 4 Gyr). At the conclusion of the simulations (4.5 Gyr) the single star model had $5\,000 M_\odot$ remaining in stars, twice that of the 50% binary model.

Initially the core density of each of the simulations decreased as mass loss from the most massive stars (stellar winds and supernovae) caused an overall expansion of the inner regions. Then, after about 100 Myr of evolution, the core of the cluster without binaries began to contract and this process continued until the maximum core density of $900 M_\odot/\mathrm{pc}^3$ was reached when the cluster was 1 400 Myr old (three half-mass relaxation times had elapsed by this point). By this time the core radius had been reduced by a factor of 10 from its original size and 7 hard binaries had formed in the core. The first binary formed after 350 Myr and the maximum number of binaries present in this simulation at any one time was 11 at 3 Gyr. After core-collapse had been averted the core experienced a slight expansion and the core density subsequently reached a fairly static, although noisy, state. In contrast, the behaviour of the core is extremely regular for the simulation with 50% primordial binaries and it is difficult to detect any sign of core-collapse.

The presence of a large fraction of binaries also has consequences for the stellar populations of star clusters. For example, Shara & Hurley (2002) used simulations of 20 000 stars and $f_\mathrm{b} = 10\%$ to demonstrate that open clusters produce supra-Chandrasekhar mass double-white dwarf binaries with merger timescales less than a Hubble time at a greatly enhanced rate relative to the field. Orbital hardening of existing binaries and exchange interactions that produced new binaries were the mechanisms responsible for the enhancement of these type Ia supernovae candidates.

4. The White Dwarf Sequence in Star Clusters

The technique of using the white dwarf (WD) sequence in the colour-magnitude diagram (CMD) of a star cluster to derive a "cooling age" for the cluster is finally being exploited with increasing success (Hansen et al. 2002, for example). However, the accuracy of this technique relies to a large extent on being certain that the true *bottom* of the WD sequence has been reached, i.e. that the coolest WD has been observed (Brocato, Castellani & Romaniello 1999). We now investigate the morphology of the cluster WD sequence by examining the CMD at 4 Gyr for all WDs in the $N = 30\,000$ simulation with primordial binaries (Figure 2).

We start in Figure 2a by plotting only what we call standard single WDs. By this we mean that each of the WDs, and their progenitor stars, were never part of a binary or involved in a collision and have evolved according to the standard picture of single star evolution. This produces the smooth cooling track seen in Figure 2a. For these WDs it is true that the more advanced along the track that a particular WD is, the more massive and older it is. We highlight the standard single WDs because these are the objects that the cooling models used to age the WD sequence directly relate to.

Next we add in all the remaining WDs that are single at 4 Gyr but whose progenitor was originally a member of a binary (Figure 2b). Even though a substantial fraction of these WDs overlie the standard cooling track it is evident that the remainder contribute a great deal of scatter to the CMD. As an example, some of these WDs have evolved from blue stragglers, or more generally any MS star rejuvenated by mass-transfer. These progenitors' journeys to the asymptotic giant branch have been delayed, hence when the WD was born it was more massive than WDs born from standard single stars at that time. This yields WDs lying below the standard WD sequence. Conversely, WDs less massive than expected at birth are produced from giants initially in binaries that overfilled their Roche-lobe and lost their envelopes prematurely, and then lost their partners in exchange interactions.

In Figure 2c we complete the full WD CMD by including all the double-WD binaries present at 4 Gyr. The first thing to notice is that for the most part the double-WD sequence is well separated from the standard cooling track. This is because the simulated double-WD binaries are mainly high mass-ratio systems. Provided that photometric errors are modest (say, ≤ 0.5 mag) it is possible to minimize contamination of the WD sequence by double-WDs in at least the upper half of the WD CMD. Further down the sequence we notice that double-WDs clump-up at brighter magnitudes than do the single-WDs and start to approach the WD sequence. As a result, estimating the position of the bottom of the cooling track by the detection of WDs blueward of the track, or by a build-up of WDs at a certain magnitude, can be seriously misleading. It

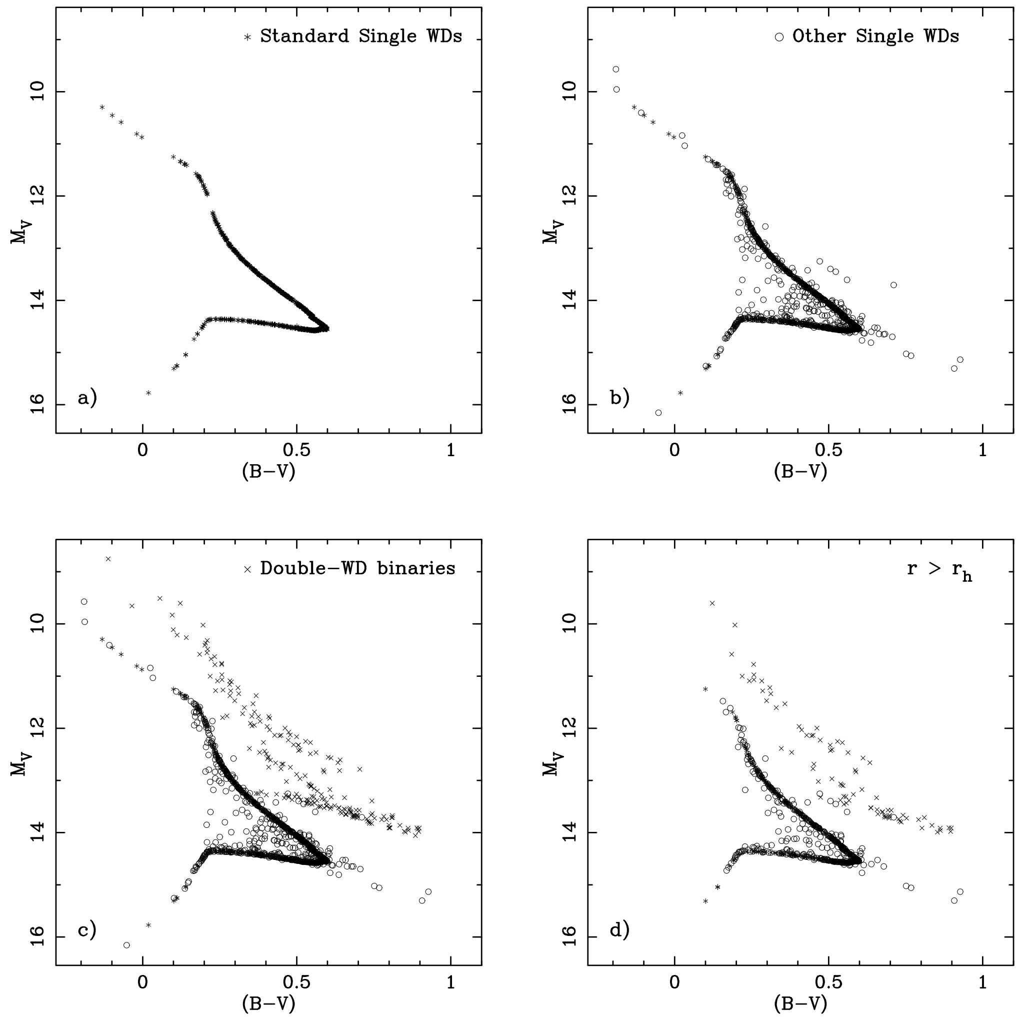

Figure 2. Cluster CMD for WDs at 4 Gyr. Stars from the N-body simulation starting with 30 000 stars and $f_{\rm b} = 50\%$ are shown. All WDs are assumed to be of DA type and bolometric corrections are taken from Bergeron et al. (1995). We have distinguished three different types of WDs depending on their binarity and formation path: single WDs that evolved from single stars (standard), single WDs for which the progenitor star (or stars) was previously the member of a binary, and double-WD binaries. Note that all binaries are assumed to be unresolved. Panel (a) shows only the standard WDs, panel (b) adds in the remaining single WDs, and panel (c) shows all three types. Panel (d) is a replica of (c) but shows only WDs that lie outside of the cluster half-mass radius (typically 4 pc).

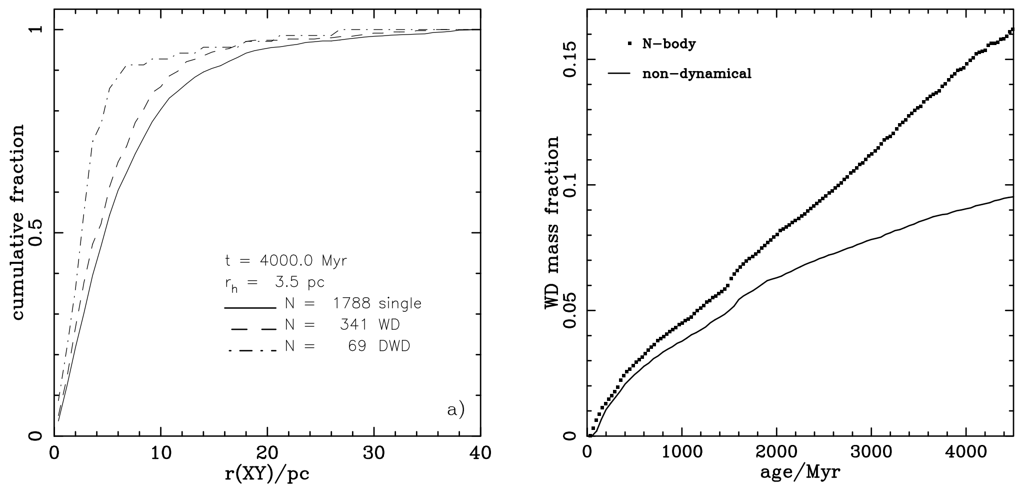

Figure 3. Left panel: cumulative radial distribution of single stars, single WDs, and double-WDs at 4 Gyr for the simulation with a 50% binary fraction. Right panel: the WD mass fraction as a function of cluster age for the same simulation (solid squares) and the corresponding mass fraction for the same population evolved outside of the N-body code (solid line).

is possible to just be seeing the scatter in the WD sequence produced by non-standard WD evolution, or by a population of old double-WDs, and one may need to go deeper to find the true extent of the track.

Outside of the half-mass radius, r_h, of a star cluster the number density of stars is less than in the core and the incidence of stellar interactions is also less. The binary fraction is also smaller in this region as mass-segregation is effective in causing binaries to sink towards the cluster centre. This is also true of double-WD binaries which have an even higher average mass than standard binaries (see left panel of Figure 3). These considerations lead to a much cleaner WD sequence, as shown in Figure 2d, and it is here that observations of WD sequences (for the purposes of age dating) in star clusters can most cleanly be conducted, if enough WDs are present.

5. The Cluster White Dwarf Mass Fraction

The determination of the WD mass fraction in a cluster is also of importance as it relates to the nature of the stellar IMF and the WD population of the Galactic Halo (von Hippel 1998). Figure 3 (right panel) shows the fraction of the cluster mass contained in WDs as a function of time for the N-body simulation with 50% primordial binaries discussed in this paper. Also shown is the WD mass fraction, $f_{\rm WD}$, for the same primordial population evolved without dynamics. It is clear that as time progresses, and the simulated cluster becomes dynamically more evolved, that the cluster environment has a significant effect on the measured WD mass fraction. More correctly it is a combination of the cluster environment and the environment that the cluster resides in that is producing this effect, i.e.

mass-segregation causes low-mass MS stars to move to the outer regions of the cluster so that these stars are preferentially stripped from the cluster relative to heavier objects such as WDs (see left panel of Figure 3). We note that some central concentration of the WD population is to be expected because their average mass is greater than that of all the cluster stars. However, the main reason for this concentration is that the progenitors of the WDs were originally more massive than the current MS turn-off mass and therefore the WDs are more likely to be born interior to $r_{\rm h}$ (see also Portegies Zwart et al. 2001).

For young open clusters, less than $\sim 3-4$ relaxation times old, the results of our simulations suggest that it is safe to assume that the WD mass fraction has been little affected by the kinematic evolution of the cluster (von Hippel 1998). However, for older open clusters and for globular clusters it would be incorrect to make this assumption (see also Vesperini & Heggie 1997). After 4 Gyr of cluster evolution, or ~ 15 half-mass relaxation times, $f_{\rm WD}$ has approximately doubled in comparison to the population evolved without dynamics. This translates to an error of $\Delta\alpha \simeq 0.2$, or 10%, in the slope of the inferred power-law IMF if the dynamical evolution is ignored. We note that the position in our simulated clusters where $f_{\rm WD}$ matches that of the non-dynamical population, after 4 Gyr, is at three half-mass radii from the cluster centre. We urge anyone utilising observations of the WD mass fraction in dynamically evolved star clusters to account for the dynamical history of the cluster.

References

Aarseth, S. J. 1999, PASP, 111, 1333

Bailyn, C. D. 1995, ARA&A, 33, 133

Bergeron, P., Wesemael, F., & Beauchamp, A. 1995, PASP, 107, 1047

Brocato, E., Castellani, V., & Romaniello, M. 1999, A&A, 345, 499

Hansen, B. M. S., et al. 2002, ApJ, 574, L155

Heggie, D. C., & Aarseth, S.J. 1992, MNRAS, 257, 513

Hurley, J. R., Pols, O.R., & Tout, C.A. 2000, MNRAS, 315, 543

Hurley, J. R., Tout, C.A., Aarseth, S.J., & Pols, O.R. 2001, MNRAS, 323, 630

Hut, P., et al. 1992, PASP, 104, 981

Kroupa, P., Tout, C. A., & Gilmore, G. 1991, MNRAS, 251, 293

Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS, 262, 545

Makino, J., & Taiji, M. 1998, Scientific Simulations with Special-Purpose Computers – the GRAPE Systems (New York: Wiley)

Meylan, G., & Heggie, D. 1997, A&A Rev., 8, 1

Portegies Zwart, S. F., McMillan, S. L. W., Hut, P., & Makino, J. 2001, MNRAS, 321, 199

Shara, M. M., & Hurley, J. R. 2002, ApJ, 571, 830

Vesperini, E., & Heggie, D. C. 1997, MNRAS, 289, 898

von Hippel, T. 1998, AJ, 115, 1536